The reviewer is aware that with all endeavor he has given but an imperfect account of this remarkable book. That Klein's researches constitute a splendid advance in the dynamics of the rotation of a rigid body there can be no question. One cannot but hope that the outline given in these Princeton lectures may soon be expanded and put in shape more easily assimilable by persons moderately versed in the theory of elliptic functions. The boon of an appropriate lemma is ideal generosity, and not even a mathematician can scorn its almost mathematical elegance. A man may be a thoroughgoing soldier enough on land; but put him in the foot ropes of the flying jibboom in a storm, and he is apt to cut a most ludicrous figure. Shift a physicist's foothold of Cartesian differential coefficients, suspend him over an abyss of non-Euclidian space, and he will kick sturdily. Poor policy this, for a missionary !

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THE TRANSMISSION OF RADIANT HEAT BY GASES AT VARYING PRESSURES.*

BEFORE describing my own investigations on the transmission of heat by gases, I shall refer briefly to the classical work of a somewhat similar nature by MM. Dulong and Petit early in the present century, 'Researches on the Measure of Temperatures, and on the Laws of Communication of Heat,' *Ann. of Phil.*, 1819.

In their researches on the 'Communication of Heat,' Dulong and Petit used as the cooling body a very large thermometer bulb filled with mercury, and as the recipient of the heat a large copper bulb or 'Balloon' about three decimeters in diameter, in the center of which the thermometer bulb was

*Abstract of a paper read before the American Association for the Advancement of Science, August 10, 1897.

placed. The copper balloon was coated with lamp-black on the inside, and kept at any desired constant temperature by means of a water-bath or melting ice. The thermometer tube was of such length as to bring the zero of the scale outside the balloon ; and the thermometer was adapted to be removed, heated and quickly replaced, air-tight. The balloon was connected with an air-pump capable of rapidly exhausting it down to about two millimeters pressure, and also with a gas-holder from which it could be quickly filled with the gas whose cooling properties were to be determined. The rate or ' Velocity' of cooling of the thermometer bulb was deduced from observations of the falling temperature at equal intervals of time.

With this apparatus Dulong and Petit made many carefully conducted experiments at differences of temperature between the thermometer and balloon ranging as high as 300 degrees; and with several different gases besides air, ranging in pressure from atmospheric to two millimeters. From the results of these experiments they deduced several laws of cooling which they held to be general in their application. They sharply divided the cooling into two parts : that due to convection—the actual contact of the surrounding cooler gas renewed by its own currents, and that due purely to radiation—the same as would occur in an 'absolute vacuum.' They derived a constant value for the latter, and values for the former varying with different gases and different pressures. They generally used the thermometer bulb naked, with its natural vitreous surface, but sometimes they silvered it. While this radical change in the character of surface greatly changed the loss of heat due to radiation, it apparently had no effect on that due to convection.

MM. Dulong and Petit fell into the grave error of deducing the behavior of the last

few millimeters of gas from that of the rest. In this way they arrived at the following 'Sixth Law :'

"The cooling power of a fluid diminishes in a geometrical progression when its tension itself diminishes in a geometrical progression. If the ratio of this second progression is 2, the ratio of the first is 1.366 for air; 1.301 for hydrogen ; 1.431 for carbonic acid, and 1.415 for olefiant gas."

My own observations show that this law can be approximately true only in the case of a large balloon, and at pressures from a few millimeters upward. There is no suggestion of it when a small balloon is used, and at small pressures it does not obtain with either large or small balloons.

It was through misplaced confidence in their Sixth Law that Dulong and Petit were led to place a value on the rate or velocity of cooling in vacuo, something like a hundred per cent. too high, and as they derived the cooling values of gases by deducting the cooling effect of a vacuum from the total cooling observed, all their values for gases are much too low.

Other experimentalists, also, have studied the transfer of heat by air and other gases at various pressures. Kundt and Warburg (Pogg. Ann., 1874–5), and Winkelmann (Pogg. Ann., 1875–6), observed that the rate of heat transmission remained substantially constant through a long range of diminishing pressure, and then decreased with further exhaustion. But as they made no measurements of pressure below one millimeter (1316 millionths of atmospheric pressure), their results have no quantitative value for low pressures.

Crookes, in his paper, 'On Heat Conduction in Highly Rarefied Air' (Proc. Roy. Soc., 1880), described a similar experiment in which he carried the pressure measurements as low as 2M. (two millionths). From the fall in the rate of heat loss which occurred between the pressures of 760 milli-

meters and 1 millimeter, and 5 M. and 2 M., he concludes: "We may legitimately infer that each additional diminution of a millionth would produce a still greater retardation of cooling, so that in such high vacua as exist in planetary space the loss of heat —which in that case would only take place by radiation—would be exceedingly slow."

In this conclusion Mr. Crookes was, I think, wrong. I find that the curve representing the rate of cooling does not break down materially at pressures as low as a twentieth of a millionth.

My own investigations on 'The Transmission of Radiant Heat by Gases at Varying Pressures' form a part of a general study of the properties of high vacua, in which I have long been engaged.

In the course of my work it became necessary to know how much of the heat communicated by a good radiating body at ordinary temperatures, to a neighboring body at a slighly lower temperature, through an intervening gas, is transmitted by the so-called ether, and how much by the gas ; and whether any of that transmitted by the gas is communicated otherwise than by the process of convection. Also why, and to what extent, do the gases differ from each other in their heat transmitting capacities.

In the drawings herewith, Fig. 1 is a diagram of the apparatus used in my experiments. A is the thermometer whose cooling was observed. It has a very open scale divided into two-tenths degrees C. The zero point is placed a long distance (about 170 millimeters) above the bulb, for obvious reasons. The bulb is cylindrical, about 20 mm. long, and about 7 mm. in diameter, and is coated with lamp-black applied with a very thin alcoholic solution of shellac. After several hours' baking at 100 degrees in a good vacuum, this bulb gave constant radiation results. The thermometer is suspended by a platinum wire, with its bulb in the center of the large

Fig. 1.

FIG. 1.

pear-shaped glass bulb B, about 112 mm. in diameter. The stem of the thermometer hangs freely in the long neck of the large bulb. I shall hereafter call the glass bulb B the 'large radiation bulb,' or simply the 'large bulb,' to distinguish it from a smaller one used later. The bulb B is surrounded by a copper tank C, lagged with woolen cloth, and filled with crushed ice and distilled water. A wire netting C' serves to keep some of the ice always below the lowest point of B. The tank C is movable on vertical guides, whereby it may quickly be raised to, or lowered from, the position shown, thus exposing the bulb B alternately to the ice bath and the atmosphere of the laboratory. The bulb B communicates freely with the large barometer tube D, which is used for measuring all but very small pressures. E is a standard boiled barometer, dipping into the mercury cistern F, common to both barometers. G is a McLeod gauge giving very accurate measurements of small pressures, and H is a drying bulb containing phosphorous pentoxide. The glass stopcock I serves to admit other gases than air. The mercury valve K prevents any leakage backward from the pump when the latter is stopped, during observations. Exhaustion is effected by an automatic Sprengel pump having five fall tubes. L is a fine cathetometer placed in front of the whole apparatus, and by rotation on its vertical axis is adapted to read the McLeod gauge, both barometers, and the thermometer. It has a vertically divided scale with veruier and microscope, for reading the barometers, and a micrometer for reading the gauge. A watch N is mounted close beside the thermometer on a sliding frame, so as to be easily kept in the field of view of the cathetometer telescope when the latter is used to observe the falling temperature.

Before using this apparatus, I always exhausted to a good vacuum and heated the bulb B by means of a water-bath, and all other vacuous parts by means of an air bath, to 100 degrees for several hours. This was found necessary in the first instance with air, in order to divest the inner glass surfaces of that portion of their coating of adherent gas most easily given off in a vacuum. This gas was pumped out, and, not being principally air, was not largely reabsorbed when air was admitted. Without this precaution I was unable to obtain constant results at very low pressures. When other gases were tried successively, the preliminary heating prevented gas from one operation attaching itself to the glass and remaining to contaminate the succeeding gas at very low pressures.

I next introduced the proper gas up to atmospheric pressure and made a preliminary cooling of the thermometer by raising the ice tank C. This preliminary cooling was found to have a slight effect on the readings next following, and was done to make the first set of readings on any day entirely comparable with the others. I then lowered the ice tank, and, when the temperature had raised to 18 degrees, stirred the ice and water thoroughly, raised the tank again, and observed the thermometer through the telescope—noting by the watch N the instant when the falling mercury passed each degree of the scale. Then, with the ice tank still up, I noted the pressure by measuring with the cathetometer the difference in height of the barometer columns in D and E. The barometer D showed that the gas in the radiation bulb cooled nearly to zero with very great rapidity when the ice tank was raised. I always measured pressures with the radiation bulb cold. It was usual to repeat the whole operation to confirm results before reducing the pressure by the pump.

Observations were thus made at pressures varying from atmospheric down to the best vacuum obtainable. In some instances

many series of observations were made at varying pressures all within the last millionth. The gauge could be relied upon to measure these small pressures with very great accuracy ; but it was difficult to maintain them long at an exactly constant value on account of the continual, though slight, evolution of gas from the glass of the apparatus.

As I desired only comparative results, no correction was made for the probable slight inequalities in the callibration of the thermometer ; nor for heat conducted to or from the bulb by the stem ; nor for the change of zero point due to changing external pressure. The mercury fell exactly to zero at atmospheric pressure, and about one-fiftieth of a degree lower at no pressure. The pressure error due to differences of capillary depression in the two barometers was ascertained at high exhaustions, and found nearly constant. It was always corrected. The different gases used were carefully prepared and dried, and were introduced quite free from any admixture with air.

My observations have extended over a long period, and are far too voluminous to be recorded here in detail. But I have embodied their most salient features in a series of curves which render them readily apparent to the eye. In these curves the abscissae represent the pressure, and the ordinates represent the rate of heat transmission through the gas, from the thermometer bulb to the ice-cold envelope. The rate of transmission at any particular pressure is expressed by the reciprocal of the number of seconds required for the temperature to fall through a given number of degrees. For convenience of scale, all the reciprocals are multiplied by 500.

Fig. 2 shows the curve for air. The heavy line represents the rate of cooling from 15 degrees to 10 degrees. It is in three sections, A, B and C. Section A embraces the whole range of pressure from nothing to

atmospheric ; section B embraces the range of pressure from nothing to .01 of atmospheric ; and section C embraces the range of pressure from nothing to .0001 of atmospheric, i. e., 100 M. (one hundred millionths). Atmospheric pressure is taken at 760 mm. Thus it will be seen that section B is the last hundredth of A, magnified a hundred times ; and section C is the last hundredth of B, magnified a hundred times. This magnification of the abscissae without change of the ordinates, enables us to study every part of the curve with ease. The small circles represent the points in the curve established by observation. These points are shown exactly as found, without any attempt to smooth out rough places in in the curve. The same is true of the curves of other gases. The heavy dotted line parallel with the base indicates that portion of the total heat transmission due to the ether ; while all above it represents that due to the air.

Starting at the left-hand end of section A, representing the rate of heat transmission at atmospheric pressure, we observe that the curve drops regularly at a rate faster than the diminution of pressure during ninety-five per cent. of the whole range of pressure from atmospheric to zero. Beyond this point the rate of heat transmission remains substantially constant, as shown by section B and the latter part of A, down to a pressure of about .0003—a range of nearly ninety-nine and a-half per cent. of that remaining. Here the curve suddenly begins to drop again, and falls steadily, as shown by section C and the latter part of B, until it meets the ether line at the zero of pressure.

Under the curve A, I have drawn curves with finer lines, representing the rate of heat transmission at smaller differences of temperature between the thermometer and ice bath. As before stated, A represents the cooling from 15 degrees to 10 degrees.
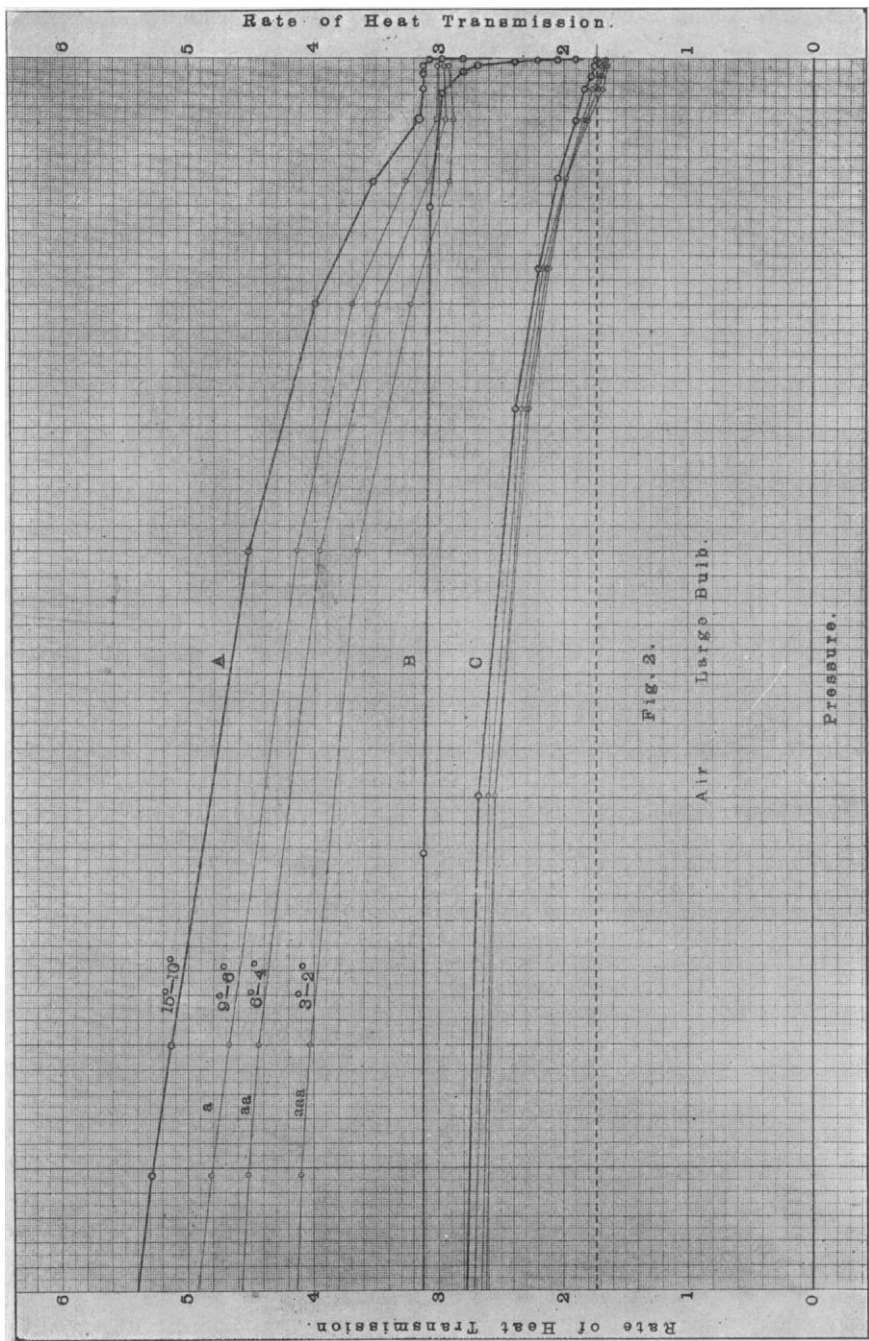
Fig. 2. Air Large Bulb.

FIG. 2.

On the same scale a represents the cooling from 9 degrees to 6 degrees; aa from 6 degrees to 4 degrees, and aaa from 3 degrees to 2 degrees. Now, Newton's law of cooling requires that the rate shall vary directly with the difference of temperature between the cooling body and the surrounding medium. While this law is known to be incorrect for large differences of temperature, it is generally accepted for very small differences. If it were correct under the conditions of the present experiment, then the ratios of the times required for the temperature to fall through the several ranges above indicated would all equal unity, and the curves A, a, aa, aaa would coalesce. But they are very far from doing this. It will be observed that all of these curves preserve their relative values very closely indeed, until they approach the point of pressure where the curve A reverses itself; then they begin to bunch themselves very much closer together, especially the lower ones, and shortly reach a greatly reduced as well as varied ratio of values which they retain substantially unchanged to the end, as shown in connection with section C. To avoid confusion of lines, I have omitted the secondary curves corresponding with section B.

Carbon monoxide was chosen for comparison with air, because its absorptive power for radiant heat is many times greater, while its specific heat is almost exactly the same. The principal curve, representing the rate of heat transmission from 15 degrees to 10 degrees, differs very little from that of air. It shows a slightly better rate than air at very small pressures; not quite so good a rate as air at intermediate pressures; and the same rate at atmospheric pressure. But the curves a, aa, aaa, representing equivalent amounts of cooling at smaller temperature differences, are materially unlike those of air. At high pressures they have about the same ratio values as with air; but the ratio diminishes much less at intermediate and low pressures; that is to say, the curves remain further apart. It is equally noticeable that the curves aa, aaa retain their full relative ratio values at low pressures, while with air they nearly coalesce.

It was thought that ethylene might transmit heat more rapidly than air, because of its much higher specific heat. But it does not do so. Its curve has the same form as those of air and carbon monoxide. It transmits heat nearly as well as air at atmospheric pressure, but not nearly so well at intermediate pressures. At a very few millionths, however, it conducts a trifle better than air. The curves a, aa and aaa have the same characteristics and about the same ratios as those of carbon monoxide.

Hydrogen was next tried, on account of its very low coefficient of viscosity, as well as its very high specific heat. While in general form the hydrogen curve resembles the air curve, all the ordinates are immensely increased. It is noticeable that the intermediate section B of the curve lies much nearer A than C, quite different from its relative position in the curves of the other gases. This section of the curve shows that hydrogen retains about two-thirds of its initial heat transmitting power at a pressure nearly two hundred times smaller than does air. The curves A, a, aa and aaa have something like the same ratios as they have in the cases of carbon monoxide and ethylene. In general, it may be said of hydrogen in the large radiation bulb, that it transmits heat nearly four times as fast as air at atmospheric pressure; more than twice as fast at a very few millionths, and more than seven times as fast through a long range of intermediate pressures.

As evidence of the accuracy of the observations on which the curves thus far described are based, it is gratifying to note
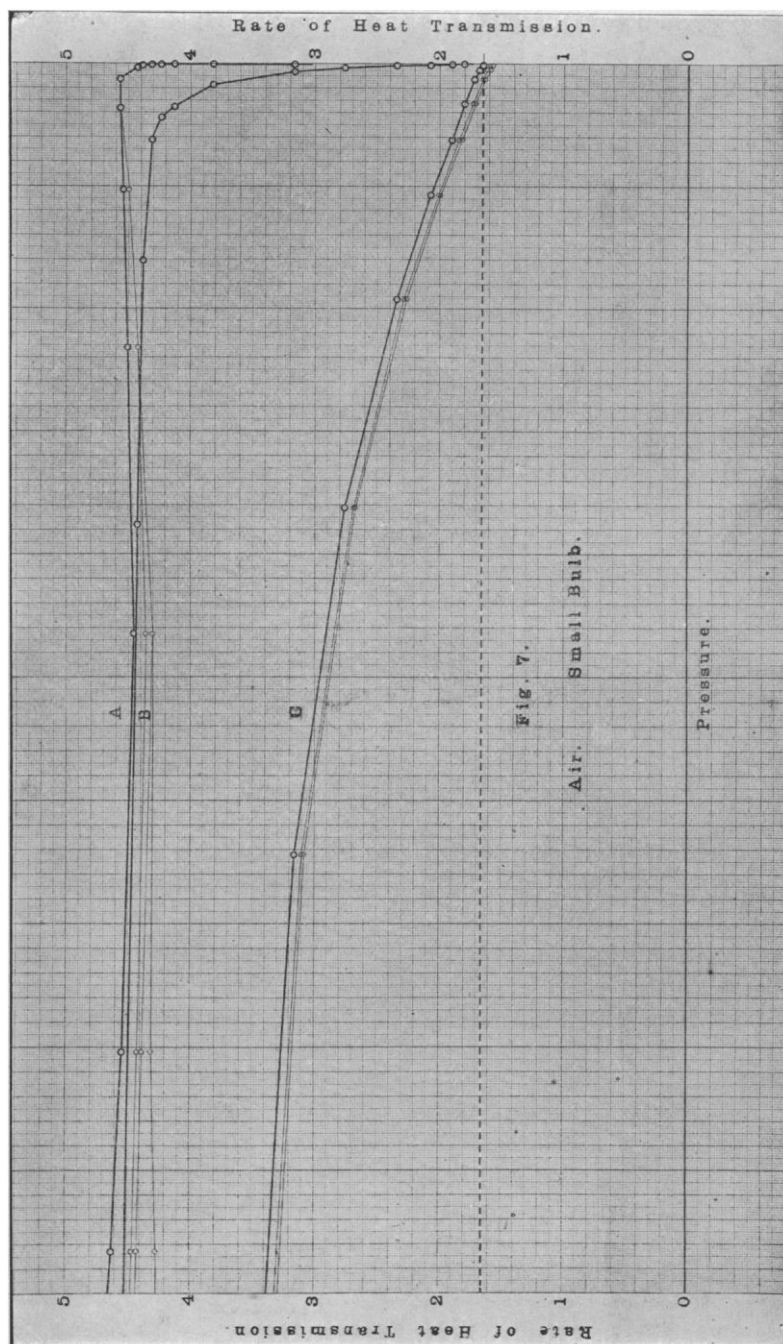
Rate of Heat Transmission.

Fig. 7. Air. Small Bulb.

Fig. 3.

that the vacuum, or ether line, locates itself exactly the same in all.

In making the above described observations, I looked for some change in the phenomena when the exhaustion reached the point at which the mean free path of the gas molecules equalled the distance between the thermometer bulb and the cold walls of the enclosing globe. This should have been at a pressure of about two millionths. No such change was observable, however, in any case. Partly in pursuance of the same idea, I resolved to repeat some of my experiments, using a very much smaller radiation bulb. This I expected would also reduce that portion of the total cooling effect due to convection currents. I accordingly employed the bulb of tube P, Fig. 1, in my further experiments. This is made from a thin glass tube slightly less than 20 millimeters internal diameter, and in it hangs the same thermometer A which was used before. In transferring the thermometer great care was taken to avoid any disturbance of the coating of lampblack on its bulb. At b is a contraction of the tube P, to prevent the thermometer bulb swinging against the inside of the tube. The contraction b is, however, much larger than the thermometer stem, so that normally the latter does not touch it. The thermometer bulb hangs exactly in the center of P, near its bottom, and is separated from it by a space of a trifle more than six millimeters—almost exactly a quarter of an inch—instead of two inches, as in the case of the 'Large bulb.' The tube or bulb P, I shall hereafter designate the 'Small radiation bulb,' or simply 'Small bulb,' to distinguish it from the large one.

The curve for hydrogen, with the small bulb, differs radically in size and form from that obtained with the large bulb. Section A, instead of drooping rapidly with decreasing pressure, maintains almost full value throughout. Section B starts with nearly

double its old value, but breaks down much earlier. Section C starts with a little higher value, but is much straighter, and consequently has a lower value throughout most of its length. The curves a, aa, aaa are very peculiar. They start at atmospheric pressure with much smaller total and very different relative ratios than before, and are successfully absorbed into A. They reappear later, however, but with small ratios.

Fig. 3 gives the curve for air, with the small bulb. It differs from that with the large bulb quite as much as did the hydrogen curve. Section A droops slightly, and regains almost its full atmospheric value at one per cent. pressure. Section B has the same form as with the large bulb (Fig. 2), but more than double its value ; and section C also has a much higher value throughout. The curves a, aa, aaa have small ratio values at the beginning, and are absorbed into section A, the same as with hydrogen. But aa and aaa coalesce when they reappear, and coincide to the end ; while the ratio between a and aa remains constant at a very small value.

The curve for carbon dioxide, with the small bulb, closely resembles the air curve in form, but has a very much smaller value throughout. While the curves aa and aaa are soon united, and remain so to the end, a and aa never disappear as they did in the cases of hydrogen and air.

With the small bulb, as with the large, no change in the character of the phenomena was observable when the exhaustion had reached the point at which the mean free path of the molecules equaled the space through which the heat was conducted. This point was reached in the small bulb at a pressure of about fourteen millionths.

It seems reasonable to assume that the radical difference between sections A of the curves obtained with the large and small bulbs respectively was due to an almost complete suppression of convection currents

in the latter case. In the absence of convection currents that part of the heat transmitted by the gas was probably carried by a process analogous to conduction in solids. The shortness of conductor in the case of the small bulb may account for the greatly increased rate of conduction. But why the conductivity of a gas remains nearly constant through a very wide range of pressures is not clear. Mr. Crookes' explanation of this phenomenon seems to me very unsatisfactory.

It will be noticed that the ' Ether line ' is about four per cent. lower with the small bulb than with the large one. This may be due to the greatly decreased amount of surface presented by the small bulb for absorption of the radiant heat.

The enormous heat-conducting capacity of gases at very small pressures is strikingly shown in all the curves. But hydrogen is preeminent in this respect. Thus, in the large bulb, hydrogen at a pressure of only twenty-six millionths of an atmosphere transmits heat as rapidly as the ether. At seventy-six millionths it equals air at atmospheric pressure ; that is to say, it does the work of nearly two hundred thousand times its weight of air.

It is remarkable that at pressures up to a few millionths, all the curves are nearly straight lines. This is especially noticeable in the small bulb curves ; showing that at these small pressures the heat-transmitting power of a gas varies directly with its amount. Hence it seems reasonably certain that if the very small fraction of a millionth of the gas examined, which remained at the end of each experiment, could have been entirely removed, the heat transmitting power of the vacuum would not have been materially diminished. It was customary at the end of the experiments with each gas to close the gauge permanently when the pressure had fallen to a tenth of a millionth or so ; and with

the capacity of the whole apparatus thus reduced, run the pump continuously from one to two hours. Several sets of observations were always made during this extreme exhaustion; and while the change in the rate of cooling of the thermometer was generally appreciable, it was always very small indeed. In my earlier experiments I took the greatest care to insure the absence of mercury vapor in the final vacuum. But the presence or absence of mercury vapor made no difference distinguishable from the errors of observation.

Of course, the best vacuum producible by a Sprengel pump still contains many thousands of millions of gas molecules per cubic centimeter. This may be regarded as a prodigiously large or exceedingly small quantity of gas, according to our point of view. While it has no apparent effect on the general heat-transmitting capacity of the vacuum, it does seem to interfere with or modify some function of the ether. This is the only explanation of certain phenomena that I can offer. I refer to the different behavior of the vacua with different residual gases, and in different sized bulbs, in the matter of adherence to, or departure from, Newton's simple law of cooling. The curves a, aa, aaa illustrate these differences in the several cases at the extreme end of section C of the principal curves. These differences are too large to be attributed to errors of observation. This is one of several reasons which lead me to suspect that at higher pressures all the gases examined interfere materially with and retard the transmission of heat by the ether. In other words, I suspect that the dotted ether line of my curve sheets should not be drawn parallel with the base, and have a constant value at all gaseous pressures, as shown, but should have a decreasing value as the gas pressure rises from zero. On this interesting phase of my subject I hope to have more to say at a future date.
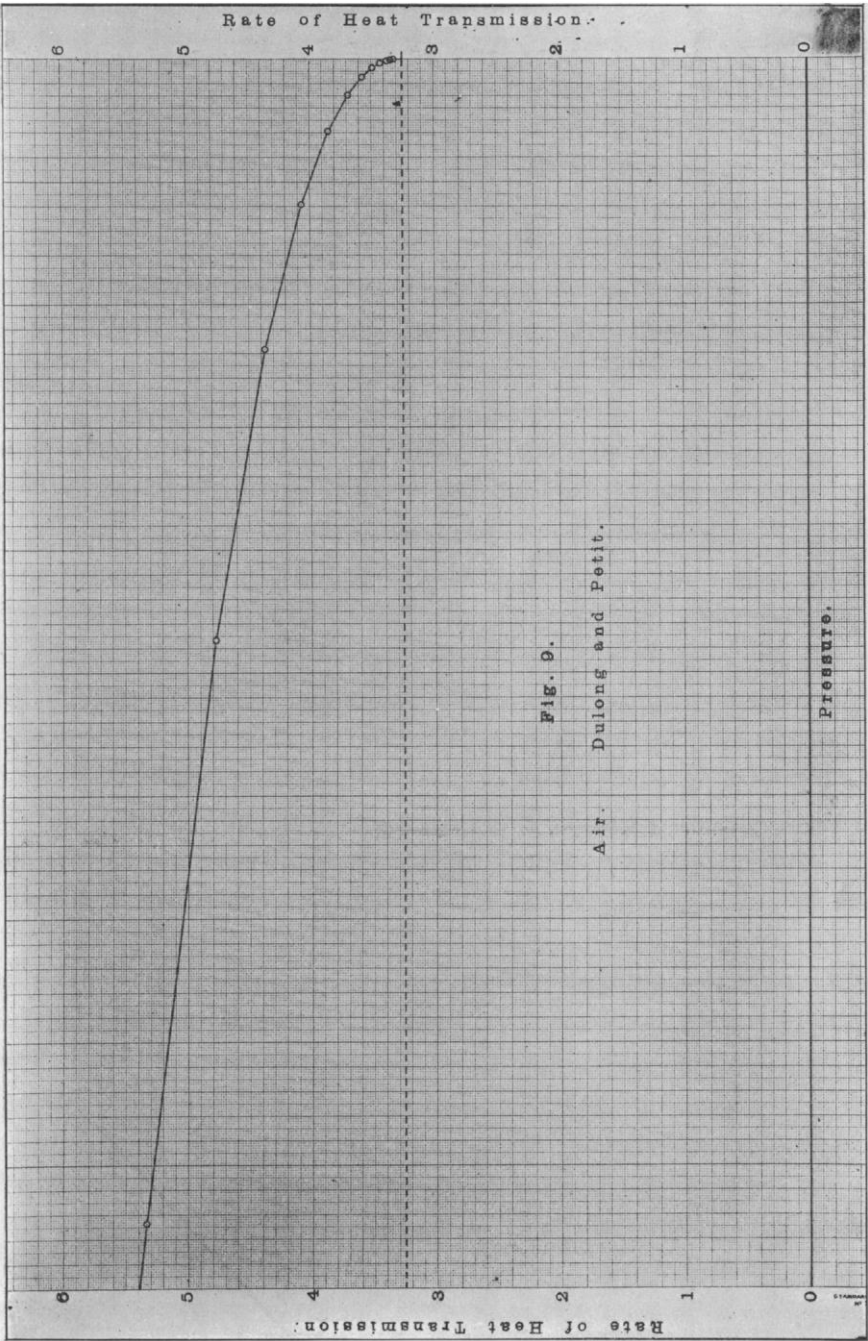
Fig. 9.

Air.   Dulong and Petit.

Pressure.

FIG. 4.

Fig. 4 is an air curve plotted from figures given in Dulong and Petit's paper. It is drawn to such a scale that the rate of heat conduction at atmospheric pressure is the same as in my own experiment with air in the large bulb, and illustrated in Fig. 2. The first five stations in the curve are the ones from which they deduced their 'Sixth Law' of cooling. The rest of the curve is drawn in accordance with that law, and the vacuum line represents exactly the value they assigned to the cooling power of an absolute vacuum. Comparison with Fig. 2 shows how much they erred in their deductions.

A study of the curve embodying the results obtained with a mixture of three volumes of hydrogen, and five volumes of carbon dioxide in a small bulb, shows that the carbon dioxide interfered very greatly with the performance of the hydrogen. Before any exhaustion was made, the hydrogen alone would have done more than three times the work of both gases. It was not until the pressure had fallen to about one hundred millionths that both gases combined, did as well as the hydrogen would have done alone. Below this pressure both gases contributed to the result.

This interference of mixed gases is a very interesting phenomenon, and seems to warrant the careful investigation which it is my intention to give it.

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THE BREEDING OF ANIMALS AT WOODS HOLL DURING THE MONTH OF MARCH, 1898.

THROUGH the courtesy of the United States Commissioner of Fish and Fisheries, several naturalists have been enabled to make use of the equipment of the Biological Station at Woods Holl during the past month, and the following notes may be of interest to those who contemplate pursuing lines of investigation at either of the marine laboratories:

The water has swarmed with animal life, and many forms rarely or never captured during the warmer months have been found in abundance. Breeding animals have yielded rare embryological material, and all forms of life have had great vitality, due probably to the low temperature of the water. The temperature of the water has ranged from 38 F. at the beginning of the month to 43 F. on the 30th. Its specific gravity has varied from 1.0232 to 1.0236.

Among vertebrates the winter flatfish (*P. americanus*) has been taken in large numbers, and spawning individuals have yielded an abundance of embryos and young. The clustered eggs of the small sculpin (*Acanthocottus æneus*) have been taken from nets and from sea-weed, and the young have been conspicuous in the *Auftrieb*. The surface towings have also yielded young of the common cod (*G. callarias*), eggs of which were hatched at the Station during the earlier portions of the month. Young cod, from one-half to three-fourths of an inch in length, have been found feeding exclusively upon Copepods, and associated with them were the somewhat larger pollock (*Pollachius virens*). The Gadidæ have also been represented by numerous adult 'frostfish' (*Microgadus tomcod*), though the breeding period of this species is in December. The young of the sand-launce (*Ammodytes americanus*), from one-half to one inch in length, and of the eel (*A. chrysypa*), from two to two and one-half inches in length, have also been taken. The pipe-fish (*Siphostoma fuscum*) was not examined, though it was found in Narragansett Bay with eggs and with young March 22, 1897.

The 'alewife' or spring herring (*Pomolobus pseudoharengus*) has begun to enter the fresh-water streams from the sea, though it has not yet begun to deposit its eggs.

Several Crustacea are already breeding. The green crab (*Carcinus granulatus*) is car-